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Multiplying (1), (2), and (3) by -3, -1, and 2 respectively, we eliminate and p, and obtain $2q = 3 \times 2533 + 4052 + 2 \times 6080$, or $q = 11{,}905\frac{1}{2}$.

Multiplying (1), (2), and (3) by -4, 4, and -1, respectively, we eliminate p and q, and obtain $2r = 4 \times 2533 - 4 \times 4052 - 6080$, or $r = -6078$.

Hence the errors in computing P, Q, and R are respectively -6080; 11905.5 and -6078.

Excellent solutions were also received from WALTER C. EELLS, J. W. CLAWSON, and the PROPOSER.

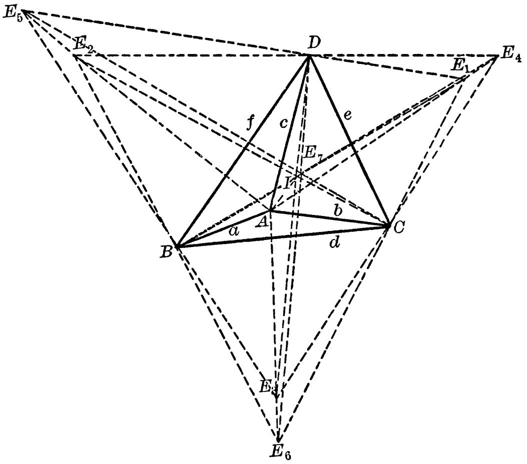
GEOMETRY.

417. Proposed by R. P. BAKER, University of Iowa.

Enumerate the points in which the twelve dihedral bisector planes of a tetrahedron meet, find their multiplicity and account for the 220 points which 12 planes in general determine.

SOLUTION BY J. W. CLAWSON, Collegeville, Pa.

Call the planes bisecting the dihedral angles internally by the letters a, b, c, d, e, f; the planes perpendicular to these which bisect the dihedral angles externally a', b', c', d', e', f'.



Six of these planes pass through the vertex A. Six planes in general determine 20 points. Now a, b, c intersect in the straight line joining A to the center of the inscribed sphere and the opposite ex-center; a', b, c' intersect in a line through A and two of the centers of escribed spheres; a, b', c' in a line joining A to another pair of ex-centers; a', b', c in another such line. These 20 points then reduce to a vertex, taken 16 times, and 4 straight lines passing through that vertex. Since there are four vertices, we can thus account for 80 of the 220 points.

Six of the 12 planes pass through the center of the inscribed sphere. Six planes in general determine 20 points. However a, b, c intersect in the straight line AI; and similarly the three planes passing through each of the other three vertices determine 3 lines. This eliminates 4 of the 20 points from consideration, since these four lines have already been considered in the paragraph above. Thus the planes intersect three by three at I, taken 16 times. Similarly each of the seven ex-centers is determined 16 times. For instance a', b', f', c, d, e' intersect at E_6. These six planes determine the point E_6 16 times and four lines AE_6, BE_6, CE_6, DE_6, already considered above. This gives us then 8 points each repeated 16 times or 128 more of the 220 points.

Twelve points remain to be located. Now e intersects a and a' at a point on AB equidistant from two faces of the tetrahedron. And e' intersects a and a' at a second point on AB equidistant from the same two faces. Similarly there are two points on each of the six edges in which three planes intersect. This gives the remaining 12 points, each determined once.

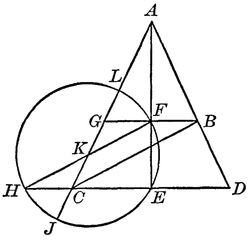
The 220 points, then, are four vertices, each 16 times; an in-center and 7 ex-centers each 16 times; 2 points on each of the 6 edges; and four straight lines from each of the four vertices to the in-center and ex-centers, these centers lying in pairs on these lines, 16 lines in all. Total 204 points and 16 lines. The figure shows these points and lines, except the 12 points, 2 on each edge.

434. Proposed by CLIFFORD N. MILLS, Bloomington, Ind.

ABC is any triangle with sides, a, b, c. Prove by purely geometrical methods that the area $A=\sqrt{s(s-a)(s-b)(s-c)}$, where $s=\frac{1}{2}(a+b+c)$.

Solution by E. E. Whitford, New York City.

Lay off $AG=AB$, draw GB, draw $CD \| GB$ meeting AB prolonged at D. Through the middle point F of GB draw AE meeting CD in E. Draw FH through K, the mid-point of GC, meeting CE prolonged at H. With K as center and KF as radius describe circle. AE is $\perp$ to GB and CD, $KF \|$ to CB and $HF=CB$. H and E are on the circumference.



$$\triangle ABC = \triangle ADC - \triangle BDC = CE \cdot AE - CE \cdot EF = CE \cdot AF.$$

By similar $\triangle$'s, $CE : AE = GF : AF$. Hence $CE \cdot AF = AE \cdot GF = AE \cdot HC$,

$$\triangle ABC = \sqrt{CE \cdot AF}\sqrt{CE \cdot AF} = \sqrt{CE \cdot AF}\sqrt{AE \cdot HC},$$

$$\triangle ABC = \sqrt{CE \cdot AF \cdot AE \cdot HC} = \sqrt{(AF \cdot AE) \cdot (CE \cdot HC)},$$

$$\triangle ABC = \sqrt{(AJ \cdot AL) \cdot (CL \cdot CJ)} = \sqrt{s(s-c)(s-a)(s-b)}.$$